modes. For $n \geqslant 2$ the cutoff wavenumber is unique and can be designated as either EH or HE.

In Figs. 2 and 3 modal loci are drawn providing the variation of $r = R_1/R_2$ versus k_2R_2 for various values of ϵ. As $r \rightarrow 1$ the cutoff values k_2R_2 tend to ∞, as expected for the case of a perfectly conducting rod.

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A Study of Waveguides for Far Infrared Interferometers Measuring Electron Density of Tokamak Plasmas

JEAN PIERRE CRENN

***Abstract*—In the 0.1–1-mm wavelength range, waveguide propagation offers some advantages over optical propagation in multichannel infrared interferometers measuring electron density of Tokamak plasmas. In this paper, the necessary conditions for use of waveguides for this purpose are defined. Possible waveguides are theoretically and experimentally studied, taking into account their shape, size, material, and length. It is shown that it is possible to find waveguides well suited for these interferometers. These results can also be applied to other far infrared interferometers and devices.**

I. Introduction

BECAUSE OF refraction effects measurement of electron density with multichannel interferometers in large Tokamaks requires the use of infrared radiation rather than microwaves [1]–[3]. For example, an eight-channel interferometer is in operation on the TFR Tokamak, using a wavelength of $\lambda = 0.337$ mm [4]. Up to now infrared interferometers have used free-space propagation of beams. However, in large Tokamaks, beam paths are several meters long, and, therefore, require the use of large optical components due to beam deviation and divergence effects which are always present. A waveguide device would be less sensitive to these undesirable effects, and moreover, would be easier to realize [5]. However, waveguide propagation must satisfy two essential conditions:

1) the propagation must avoid excessive losses,
2) the wave polarization (i.e., the direction of the electric field lines) must be linear [4].

These conditions must be satisfied even for small deviations of the beam direction at the input of the waveguide. Here we will study several waveguide structures in the 0.1–1-mm wavelength range, since this is the interesting range for interferometers used on large Tokamaks.

II. Choice of Waveguide Structure

Among the different possible waveguides, optic fibers and open or closed H guides [6]–[8] are less attractive than oversized closed waveguides (i.e., hollow waveguides with dielectric or metallic walls), for two reasons.

1) Open waveguides radiate some energy and this may result in stray signals in neighboring waveguides.

2) Attenuation in these waveguides is not as small as that of greatly oversized waveguides, because a part of the wave propagates inside a dielectric which always has some loss.

Among oversized closed waveguides, it is possible to make a distinction between those having dielectric walls and metallic walls. For most metals at microwave and far infrared wavelengths it is possible to neglect the real component of the refractive index and, therefore, consider only the imaginary component when calculating the attenuation. In the case of a dielectric, such as ordinary glass [10], it can be shown that for the lowest order mode

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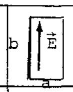
TABLE I
ATTENUATION IN METALLIC WAVEGUIDES

MODES	ATTENUATION IN SQUARE GUIDES	ATTENUATION IN TALL GUIDES b>>a	
TE_{mo} TE_{on}	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{2}{a_o}$	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{2}{b}$	
TE_{mn} $m,n\neq 0$	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{4}{a_o}$	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{4}{b}\left(1+\frac{a}{b}\frac{n^2}{m^2}\right)$	
TH_{mn}	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{4}{a_o}$	$\alpha = \sqrt{\frac{\pi\varepsilon f}{\sigma}} \cdot \frac{4}{a} \frac{1}{1+\frac{n^2}{m^2}\cdot\frac{a^2}{b^2}}$	
MODES	ATTENUATION IN CIRCULAR GUIDES		
TE_{11}	$\alpha = \frac{1,66}{D} \sqrt{\frac{\pi\varepsilon f}{\sigma}}$		
TH_{mn}	$\alpha = \frac{4}{D} \sqrt{\frac{\pi\varepsilon f}{\sigma}}$		
TE_{mn} $m\neq 0$	$\alpha = \frac{4}{D} \sqrt{\frac{\pi\varepsilon f}{\sigma}} \frac{m^2}{\xi'^2_{mn} - m^2}$		
TE_{om}	$\alpha = \frac{4}{D} \sqrt{\frac{\pi\varepsilon f}{\sigma}} \left(\frac{\lambda\, \xi'_{on}}{\pi D}\right)^2$		

a_o Side of square guide D Diameter of circular guide
ε Dielectric constant σ Conductivity
f Frequency
ξ'_{mn} n th root of Bessel function J'_m (x)

TABLE II
ATTENUATION IN HOLLOW DIELECTRIC WAVEGUIDES

MODES	ATTENUATION IN RECTANGULAR GUIDES
$E^yH^x_{mn}$	$\alpha_{mn} = \lambda^2 \left[\frac{m^2}{a^3} \frac{1}{\sqrt{V^2-1}} + \frac{n^2}{b^3} \frac{V^2}{\sqrt{V^2-1}} \right]$
MODES	ATTENUATION IN SQUARE GUIDES
$E^yH^x_{11}$	$\alpha = \frac{\lambda^2}{a_o^3} \cdot \frac{V^2+1}{\sqrt{V^2-1}}$
$E^yH^x_{mn}$	$\alpha = \frac{\lambda^2}{a_o^3} \cdot \frac{m^2+n^2V^2}{\sqrt{V^2-1}}$
MODES	ATTENUATION IN CIRCULAR GUIDES
EH_{11}	$\alpha = 8\left(\frac{u_{11}}{2\pi}\right)^2 \frac{\lambda^2}{D^3} \frac{V^2+1}{\sqrt{V^2-1}}$
EH_{nm}	$\alpha = 8\left(\frac{u_{nm}}{2\pi}\right)^2 \frac{\lambda^2}{D^3} \frac{V^2+1}{\sqrt{V^2-1}}$
E_{om}	$\alpha = 16\left(\frac{u_{om}}{2\pi}\right)^2 \frac{\lambda^2}{D^3} \frac{V^2}{\sqrt{V^2-1}}$
H_{om}	$\alpha = 16\left(\frac{u_{om}}{2\pi}\right)^2 \frac{\lambda^2}{D^3} \frac{1}{\sqrt{V^2-1}}$

u_{nm} m^{th} root of Bessel function J_{n-1} (x)

in a waveguide, the effect of the imaginary component is negligible compared with the real part [5].

Therefore, in this paper, waveguide materials will be assumed to be perfect conductors or perfect dielectrics.

Attenuation decreases in all waveguides as the transverse size increases. Thus, greatly oversize hollow waveguides appear to be the most attractive in the infrared range, in spite of inconvenience due to higher mode propagation. In the following sections, we shall study the properties of oversized hollow metallic or dielectric-wall waveguides, of simple shapes, i.e., circular, rectangular or squared.

III. Attenuation of Waveguide Modes

All the relations giving the attenuation are deduced from the general attenuation equations and are limited to the case of oversized hollow waveguides and to modes with indexes m and n not too large. These equations are well known for the case of metallic waveguides and have recently been derived for the case of dielectric waveguides [9], [11], [12] (Tables I and II). The attenuation α is defined by the relation $P_2 = P_1 e^{-\alpha z}$, P_1 and P_2 being the beam power.

For all guides, the lowest order is the most interesting to satisfy the condition of linear polarization. However, it should be noted that for hollow dielectric waveguides and especially for metallic circular waveguides, there are deformations of the electric field lines near the surface of the guide, due to boundary conditions. Computing mode attenuations (Tables I and II), it is possible to show that the lowest order mode with linear polarization has the lowest losses in the case of hollow rectangular or squared metallic or dielectric waveguides. But in the case of circular metallic waveguides, there are some modes TE_{0n}, with circular polarization and some more complex modes TE_{mn}, which have lower losses than the lowest order linearly polarized mode TE_{11}. In the case of hollow dielectric circular waveguide, the EH_{11} mode with linear polarization and the TE_{01} mode with circular polarization have roughly the same attenuation when the refractive index V is close to 2 [9].

IV. Comparison of Lowest Order Modes of Various Waveguides

It is interesting to compare the attenuation of the lowest order modes, for different shapes and materials of the guides.

In the case of metallic guides, Table I shows that for a given value of the cross section area, the circular waveguide is slightly better than the square waveguide, but the attenuation can be reduced by using a tall waveguide ($b \gg a$). For hollow dielectric waveguides, Table II shows

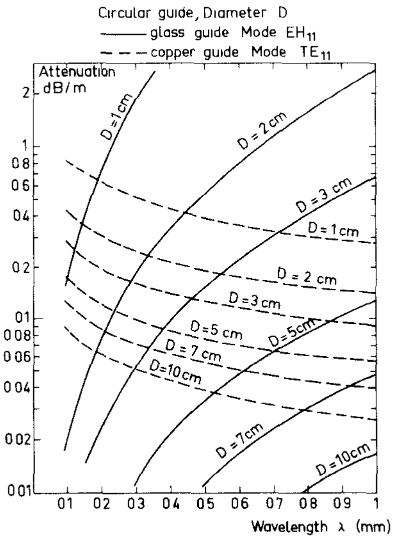


Fig. 1. Attenuation of circular guides versus wavelength.

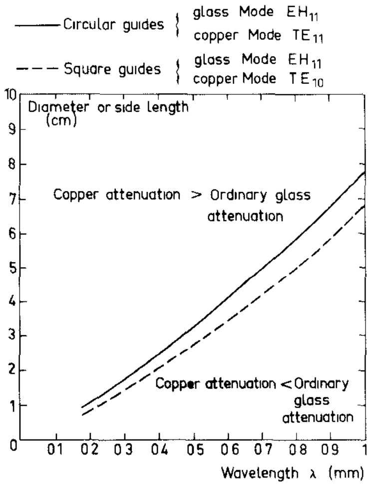


Fig. 2. Curves of equal attenuation for copper and ordinary glass waveguides.

that for a given cross section the shape of the guide does not significantly affect the attenuation.

On the other hand, Tables I and II show that the attenuation varies strongly with the frequency and with the transverse size of the guide for hollow dielectric guides, but only weakly in the case of metallic guides.

For square or circular hollow dielectric guides, the attenuation varies gradually with V and has a minimum value when $V=\sqrt{3}$. But the choice of a dielectric material with $V=\sqrt{3}$ at $\lambda=0.337$ mm, instead of ordinary glass, would lower the attenuation by only 15 percent. Fig. 1 shows a set of curves giving the attenuation of copper and ordinary glass waveguides versus wavelength, for different transverse sizes, and for circular guides (see also [13], [14]). In these curves, variation of V between $\lambda=0,1$ mm and $\lambda=1$ mm [10], are taken into account with the reasonable approximation that V is real. These theoretical curves show that, whatever the shape of the guide:

1) for weakly oversized guides, the attenuation is lower for a metallic guide than for a hollow dielectric guide;
2) for very oversized guides, the opposite is true, i.e., hollow dielectric guides are better than metallic.

It is possible to plot curves bounding the regions corresponding to these cases (Fig. 2.) Square guides are also considered.

These results can be applied to laser cavities, for instance. The cavity having the lowest losses is built with metal or dielectric according to its transverses dimension. This result leads to the choice of ordinary glass in the HCN laser cavities when the diameter is larger than 2 cm considering only the propagation losses [15]–[17].

V. Experimental Results

The beam used for the measurements is generated by an HCN laser operating at $\lambda=0.337$ mm. It has been shown that the output beam follows Gaussian beam propagation laws, in the far-field region, if a suitable mode is excited in the laser cavity [18]–[20]. The beam diameter $d(z)$ is defined in this paper as the total width at $1/e$ of the radial power distribution. The beam, during its propagation, goes through a minimum diameter d_0, called "waist".

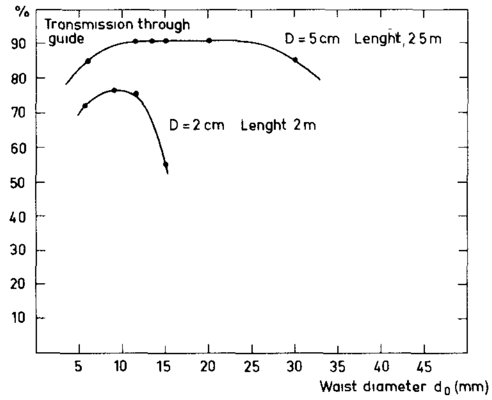


Fig. 3. Beam to guide matching: determination of the optimum waist diameter for cylindrical glass waveguides with waist at the input of the guide.

A. Conditions for Matching the Beam to the Waveguide

The problem of matching the beam to the waveguide is to find the optimum position and size of the waist such that the maximum power is transmitted into the guide. The size and position of the waist are set by concave mirrors suitably placed between the laser and the guide. It is also possible to move the guide along the beam provided that the alignment is retained. Measurements have been carried out with hollow ordinary glass waveguides of 2-cm and 5-cm diameter, and of 2-m and 2.5-m length. The beam power is measured by a calorimeter.

Results of the measurements are given on Fig. 3. The transmission given by the curves is the ratio of the output beam power to the total beam power before entering the guide. It should also be noted that in the case of a guide with 5-cm diameter, the beam distribution is almost Gaussian over some distance along the guide, as long as its diameter is smaller than that of the guide. Therefore, by changing the length of the guide, the shape of the curves would also slightly change. However, with the length used, it is possible to determine the optimum waist. This is further verified by the fact that in Fig. 3, the curves corresponding to guides of 2-cm and 5-cm diameter and with approximately the same length give very close results for the optimum value d_0/D.

A Gaussian beam was found to be well matched to the guide when the following conditions were satisfied.

1) The waist is at the input of the guide. It can be shown that this position is not critical for large diameters of the waist.

2) The diameter of the waist for the 2-cm diameter guide is in the range $0.35 < d_0/D < 0.55$, and for the 5-cm diameter guide, in the range $0.2 < d_0/D < 0.5$.

These results are in good agreement with the optimum theoretical value $d_0/D = 0.45$ found for a hollow dielectric circular waveguide [19], [21]. Using a metallic waveguide it might be thought that the results would be better in the case of a narrow waist. Indeed, a beam with a narrow waist, being very divergent, is, therefore, reflected more by metallic walls than by dielectric walls. On the other hand, with large diameter waist, the results would be approximately the same, depending only on the aperture size.

B. Measurement of Transmission Versus Incident Angle.

The incident angle i is defined as the angle between the beam axis and the guide axis (Fig. 4). The plane mirror, used to set this angle i is about 30 cm away from the guide aperture. The diameter of the waist d_0 at the input of the guide is approximately 20 mm. The power and polarization of the beam are measured at the input and output of the guide as a function of the angle i. Three guides are used: an ordinary glass guide of 5-cm inner diameter, a brass guide of 4.6-cm inner diameter, and a square brass guide with side length 3.8 cm. The transmission is defined here as the ratio between the power at the output or the guide to the power which goes into the guide. The error bars are of the order of 2 percent. The beam polarization is measured at the output of the guide, using parallel wire grids. The transmitted power varies with the orientation of the grid between a maximum value P_M and a minimum value P_m. Before going into the guide, the measured polarization is linear: $P_m/P_M < 1$ percent (± 1 percent correspond, in fact, to the accuracy of the measurements).

Results are shown in Fig. 5. The transmission without guide is also plotted on this figure. This measurement was carried out by setting a limiter of 5-cm diameter where the output of the guide would normally be situated. Comparison with the waveguide results, shows that in the case of free-space propagation, there is an important loss, even when $i = 0$. This loss is due to the beam divergence. In the case of a hollow glass waveguide, and with a horizontal displacement of the beam, transmission is better with

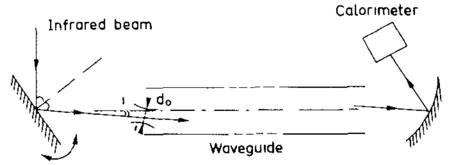


Fig. 4. Set-up for transmission measurements versus incident angle i (propagation beam plane is the horizontal plane).

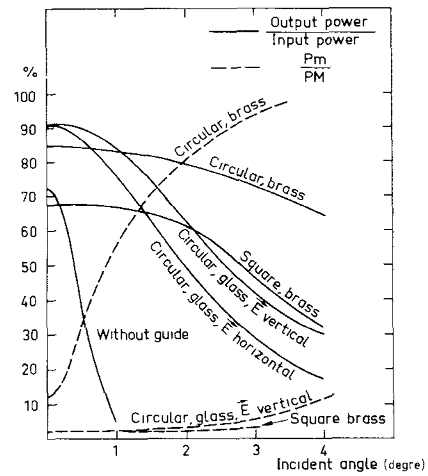


Fig. 5. Transmission and polarization measurements versus incident angle for various waveguides.

vertical beam polarization than with horizontal polarization. In the case of metallic guides, copper waveguides should be used since their attenuation is theoretically half that of brass guides. Moreover, it was found that when $i \neq 0$ there is an angle $i' \neq 0$ at the output from the guide.

Finally, these results show that in the case of a null or small incident angle i, it is better to use hollow glass waveguides. When the angle i is not small, it is better to use a metallic square guide having good conductivity. Circular metallic waveguides are not suitable because the linear polarization is destroyed by higher modes unless the angle i is zero. From the theoretical relations (Tables I and II) it can be seen that a hollow square glass waveguide would not significantly alter the attenuation and polarization as compared to a hollow circular glass waveguide.

VI. Conclusion: Application to an Infrared Interferometer on a Tokamak

This work shows that it is possible to use greatly oversized waveguides in a multichannel infrared Tokamak interferometer in order to minimize beam deviation and divergence problems in free space. Hollow circular dielectric waveguides or metallic square waveguides would be chosen depending on the transverse size of the guide, its length and the possible deviation. It is also necessary to look at some components such as bends, couplers, and mixers.

At this time these components are studied in devices close to those generally used in oversized microwave device.

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An Exact Three-Dimensional Field Theory for a Class of Cyclic *H*-Plane Waveguide Junction

RAY J. COPPLESTONE

***Abstract*—The device analyzed consists of g waveguides meeting in a cavity with a central metal disk. A conducting boundary such as this occurs in practical waveguide-junction circulators and the technique developed here may find application in circulator field theory. A special case of the geometry considered here is the 'tuning screw' which arises when $g=2$.**

The method of analysis is by representing the fields by mode summation, in the usual way, and then matching to the metal surfaces and across various imaginary internal boundaries. The device is assumed lossless.

The agreement between experimental and theoretical results is very good, thus indicating the method is valid and has been formulated correctly.

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I. Introduction

UNTIL RECENTLY there had been little work published regarding complete three-dimensional field theories of waveguide junctions with application to circulator geometry. Davies [1] in 1962 and El-Shandwily *et al.* [2] in 1973 produced theories for *H*-plane circulators with variation confined to the *H*-plane. Recently a method was formulated for handling some of the three-dimensional problems occurring in an *E*-plane waveguide-junction circulator geometry [3].

An *H*-plane circulator design may assume the form of a full height ferrite cylinder standing on a metal disk